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TITLE– On the Translational Motion of a Telescope Elastically Coupled to a Spacecraft Carrier

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ABSTRACT

The equations of motion for a spacecraft carrier to which a telescope is elastically coupled are presented in the form of system matrices. From these a transfer function is obtained for the translational motion of the telescope relative to the carrier. This motion is evaluated under disturbances from crew motions within the carrier and the external torques and forces which act on the configuration during its motion in orbit. It is assumed that the carrier has a CMG attitude control system which keeps the configuration inertially stabilized with respect to the external torques, i.e., essentially the gravity gradient and aerodynamic torques.

It is shown that the maximum relative translational motion of the telescope is a direct function of the Eigenfrequency of the telescope's suspension system. Further, the Eigenfrequency of the suspension system should be chosen, in general, to be larger than the frequency of the orbital motion of the space telescope carrier and less than the lowest elastic Eigenfrequency of the telescope structure.

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SUBJECT: On the Translational Motion of a Telescope Elastically Coupled to a Spacecraft Carrier - Case 620

DATE: June 12, 1969

FROM: J. W. Schindelin
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TECHNICAL MEMORANDUM

1. INTRODUCTION

In a previous paper [1], the influence of telescope suspension and crew motion on the equations of motion of space telescope carriers has been investigated. A companion paper [2] will present linearized expressions, system matrices, for these equations of motion. This memorandum now is concerned with a system which is basically listed as Case III in [1] and [2]. It consists of a carrier vehicle to which a mass, the space telescope, is elastically coupled in such a way that the mass has one degree of freedom for its translational motion relative to the carrier. The coupling or suspension system is characterized by a spring constant and a damping term. The main objective of this memorandum is to establish the influence which crew motion and orbital motion, i.e., the external forces which act on the system during its motion in orbit, have on the translational motion of the telescope relative to the carrier. It is assumed that the carrier has an attitude control system, e.g., control moment gyros, which keeps the configuration always inertially stabilized with respect to the external torques, i.e., the attitude control system eliminates the influence of gravity gradient and aerodynamic torque on the attitude motion of the cluster about its center of mass.

2. EQUATIONS OF MOTION, SYSTEM MATRICES

The exact equations of motion for this system are presented in [1], Section 3.3, as Case III. In [2], the corresponding system matrix for the linearized system is derived.

It is recalled here that the linearization was performed for the following state of motion [2]+) - index 0 -

$$\begin{pmatrix} \Omega_{x0} \\ \Omega_{y0} \\ \Omega_{z0} \end{pmatrix} = 0 \,, \qquad \begin{pmatrix} \Omega^T_{x0} \\ \Omega^T_{y0} \\ \Omega^T_{z0} \end{pmatrix} = 0 \,, \qquad \begin{pmatrix} \dot{R}^m_{x0} \\ \dot{R}^m_{y0} \\ \dot{R}^m_{z0} \end{pmatrix} = 0 \,,$$

$$(R^m_0) = \begin{pmatrix} R^m_{x0} \\ R^m_{y0} \\ R^m_{z0} \end{pmatrix}$$

with the additional assumptions that the vectors $\vec{P}$ and $\vec{Q}$ remain constant, i.e., with respect to the coordinate systems to which they are referred, and also (I^T) is diagonal, namely

$$(I^T) = \begin{pmatrix} I^T_{xx} & 0 & 0 \\ 0 & I^T_{yy} & 0 \\ 0 & 0 & I^T_{zz} \end{pmatrix}$$

+) Also, the acceleration terms of these state variables are zero.

The following expression was obtained for the linearized system [2], Case III+)

$$\begin{bmatrix} N_x^T - \frac{M^T}{M^T + M^b}\left[O_y^T F_{ez} - O_z^T F_{ey}\right] \\ N_z^T - \frac{M^T}{M^T + M^b}\left[O_x^T F_{ey} - O_y^T F_{ez}\right] \\ N_{ex} \\ N_{ey} \\ N_{ez} \\ F_x^T - \frac{M^T}{M^T + M^b} F_{ex} \\ F_y^T - \frac{M^T}{M^T + M^b} F_{ey} \\ F_z^T - \frac{M^T}{M^T + M^b} F_{ez} \end{bmatrix} = \left((M^{II}) + (M^{III})\right) \begin{bmatrix} \dot{\Omega}_x \\ \dot{\Omega}_y \\ \dot{\Omega}_z \\ \ddot{B}_x \\ \ddot{B}_z \\ \ddot{R}_x^m \\ \ddot{R}_y^m \\ \ddot{R}_z^m \end{bmatrix}$$

(1)

+) $\Omega_x^T \equiv \dot{B}_x$, $\Omega_z^T \equiv \dot{B}_z$. M^b includes here mass of carrier, masses of crew and CMGs.

Now, for the investigations in this memorandum, the following is assumed, see Figure 1:

$$\vec{Q} = \vec{Q}_x + \vec{Q}_y , \quad \vec{P} = \vec{P}_x + \vec{P}_y , \quad \vec{\delta}^T = \vec{\delta}^T_y$$

$$\vec{R}^m_0 = \vec{R}^m_{x0} + \vec{R}^m_{y0}$$

(I^b) diagonal. There are no servo torques about the gimbals of the telescope joint. The CMGs of the attitude control system eliminate at any time the influence of the external torques on the attitude motion of the total cluster.

Referring to [1], Section 1 and Section 4.1, Eqt. (21) with the additional term $\vec{N}_{CMG}$ introduced, the equation for the attitude motion of the total cluster about its center of mass can be written

$$(2) \qquad \vec{N}_e = \vec{N}_c + \vec{N}_T + \vec{N}_{CMG} + \frac{d}{dt} [I^b \cdot \vec{\Omega}]$$

where $\vec{N}_{CMG}$ represents the torque which is exerted by the CMGs. Because of the assumption

$$\vec{N}_e = \vec{N}_{CMG}$$

it follows for the attitude motion

$$(3) \qquad - \vec{N}_c = \vec{N}_T + \frac{d}{dt} (I^b \cdot \vec{\Omega})$$

That means, the only disturbance input in (3) is due to crew motion.

The elastic mount is given by a suspension system which is characterized by a speed proportional damping coefficient C_1 and the spring constant C_0. Furthermore, certain terms of minor influence are omitted from (M^{II}) and (M^{III}). Finally, the elastic mount shall only be able to move in Y^b direction, i.e., $\ddot{R}_x^m = 0$, $\ddot{R}_z^m = 0$ in (1).

With these assumptions, one obtains for the system matrix $(M^{II}) + (M^{III})$ the expression +)

$$(M^{II}) + (M^{III}) = (M)$$

where (M) =

$$\begin{bmatrix} I_{xx}^T + M^T O_y^T Q_y^* & -M^T O_y^T Q_x^* & 0 & I_{xx}^T & 0 & 0 \\ 0 & 0 & I_{zz}^T + M^T O_y^T Q_y^* & 0 & I_{zz}^T & 0 \\ I_{xx}^b & 0 & 0 & I_{xx}^T + M^T O_y^T Q_y^* & 0 & 0 \\ 0 & I_{yy}^b & 0 & -M^T O_y^T Q_x^* & 0 & 0 \\ 0 & 0 & I_{zz}^b & 0 & I_{zz}^T + M^T O_y^T Q_y^* & M^T E_x \\ 0 & 0 & M^T E_x & 0 & 0 & M^T \end{bmatrix}$$

(4)

+) $Q_i^* = P_i + Q_i$, $i = x,y$, $E_x = P_x + Q_x + R_{x0}^m$

System Matrix (M), (4), separates into the following subsystems which are represented in (5) and (6) with the corresponding vectors for the inputs and the state variables

$$\begin{bmatrix} N_x^T - \dfrac{M^T}{M^T + M^b} O_y^T F_{ez} \\ N_{ex} \\ N_{ey} \end{bmatrix} = \begin{bmatrix} I_{xx}^T + M^T O_y^T Q_y^* & -M^T O_y^T Q_x^* & I_{xx}^T \\ I_{xx}^b & 0 & I_{xx}^T + M^T O_y^T Q_y^* \\ 0 & I_{yy}^b & -M^T O_y^T Q_x^* \end{bmatrix} \begin{bmatrix} \dot{\Omega}_x \\ \dot{\Omega}_y \\ \ddot{B}_x \end{bmatrix}$$

(5)

$$\begin{bmatrix} N_z^T - \dfrac{M^T}{M^T + M^b} O_y^T F_{ex} \\ N_{ez} \\ F_y^T - \dfrac{M^T}{M^T + M^b} F_{ey} \end{bmatrix} = \begin{bmatrix} I_{zz}^T + M^T O_y^T Q_y^* & I_{zz}^T & 0 \\ I_{zz}^b & I_{zz}^T + M^T O_y^T Q_y^* & M^T E_x \\ M^T E_x & 0 & M^T \end{bmatrix} \begin{bmatrix} \dot{\Omega}_z \\ \ddot{B}_z \\ \ddot{R}_y^m \end{bmatrix}$$

(6)

In (6) F_y^T is the respective component of the total external force, including the suspension force, which acts on the telescope. One can write

$$F_y^T = F_y^s + F_{ey}^T$$

$$F_{ey} = F_{ey}^T + F_{ey}^b$$

where

F_y^s component due to suspension force

F_{ey}^b component acting on carrier vehicle

The component F_y^s can be expressed as

(7) $$F_y^s = -(C_0 R_y^m + C_1 \dot{R}_y^m)$$

Thus

(8) $$F_y^T - \frac{M^T}{M^T + M^b} F_{ey} = -(C_0 R_y^m + C_1 \dot{R}_y^m) + \frac{M^b F_{ey}^T - M^T F_{ey}^b}{M^T + M^b}$$

The Laplace Transformation of (6) with the substitution (8) yields

$$\begin{bmatrix} n_z^T(s) + \dfrac{M^T}{M^T + M^b} O_y^T f_{ex}(s) \\ n_{ez}(s) \\ \dfrac{M^b f_{ey}^T(s) - M^T f_{ey}^b(s)}{M^T + M^b} \end{bmatrix} =$$

$$\begin{bmatrix} (I_{zz}^T + M^T O_y^T Q_y^*)s & I_{zz}^T s & 0 \\ I_{zz}^b s & (I_{zz}^T + M^T O_y^T Q_y^*)s & M^T E_x s^2 \\ M^T E_x s & 0 & M^T s^2 + C_1 s + C_0 \end{bmatrix} \begin{bmatrix} \omega_z(s) \\ \dot{\beta}_z(s) \\ r_y^m(s) \end{bmatrix}$$

(9)

Equation (9) is the basic expression for the following investigations of this memorandum

3. DERIVATION OF TRANSFER FUNCTIONS

From (9), the following expression for the characteristic equation is obtained if $I^T_{zz} I^b_{zz} >> (I^T_{zz} + M^T O^T_y Q^*_y)$ +)

$$- s^2 I^T_{zz} A_2 \left[s^2 + \frac{A_1 s}{A_2} + \frac{A_0}{A_2} \right] = 0 \tag{10}$$

where

$$A_2 = M^T [I^b_{zz} - M^T (E_x)^2]$$

$$A_1 = I^b_{zz} C_1$$

$$A_0 = I^b_{zz} C_0$$

+) Otherwise the characteristic equation is given by the expression

$$s^2 A^*_2 [s^2 + \frac{A^*_1}{A^*_2} s + \frac{A^*_0}{A^*_2}] = 0$$

where

$$A^*_2 = (I^T_{zz} + M^T O^T_y Q^*_y)^2 + I^T_{zz} (M^T E_x)^2 - I^T_{zz} I^b_{zz} M^T$$

$$A^*_1 = C_1 [(I^T_{zz} + M^T O^T_y Q^*_y)^2 - I^T_{zz} I^b_{zz}]$$

$$A^*_0 = C_0 [(I^T_{zz} + M^T O^T_y Q^*_y)^2 - I^T_{zz} I^b_{zz}]$$

For the case

$$I_{zz}^{b} \gg M^{T}(E_{x})^{2}$$

expression (10) reduces to

$$(11) \qquad -s^{2} I_{zz}^{b} I_{zz}^{T} M^{T} \left[s^{2} + \frac{C_{1}}{M^{T}} s + \frac{C_{0}}{M^{T}} \right] \quad {}^{+)}$$

The transfer functions for the variables $\omega_z(s)$, $\dot{\beta}_z(s)$ and $r_y^m(s)$, respectively, can be obtained in the usual way from (9). Here the transfer function for $r_y^m(s)$ is of interest. It is

$$r_y^m(s) = \frac{I_{zz}^{b}}{A_2[s^2 + \frac{A_1}{A_2} s + \frac{A_0}{A_2}](M^b + M^T)} \left[M^b f_{ey}^T(s) - M^T f_{ey}^b(s) \right]$$

$$(13) \qquad - \frac{M^T E_x}{A_2[s^2 + \frac{A_1}{A_2} s + \frac{A_0}{A_2}]} n_{ez}(s)$$

$$+ \frac{(I_{zz}^T + M^T O_y^T Q_y^*) M^T E_x}{I_{zz}^T A_2[s^2 + \frac{A_1}{A_2} s + \frac{A_0}{A_2}]} \left[n_z^T(s) + \frac{M^T O_y^T}{M^T + M^b} f_{ex}(s) \right]$$

+) Eqt. (11) yields always stable roots, i.e., negative real parts, whereas (10) has, $M^T(E_x)^2 > I_{zz}^b$, roots with positive real parts.

4. RESPONSE TO CREW MOTION

For the evaluation of crew motion, see [1], Section 4. According to (3), $n_{ez}(s)$ is given by

$$(14) \qquad n_{ez}(s) = - n_{cz}(s)$$

The evaluation of crew motion response is made under the assumption that a crew member moves across the workshop with constant speed, see Figure 2, from station 1 to station 2 or vice versa. The crew member is represented by the mass M^c and its position on the X^bY^b coordinate plane by the vector $\vec{Q}^c$, (components $\vec{Q}_y^c$ and $\vec{Q}_x^c$). $\vec{Q}_x^c$ is supposed to be constant, whereas $\vec{Q}_y^c$ is a function of time, with $\dot{Q}_y^c$ being constant. That means when M^c is leaving a station, it is imparted instananeously this velocity and, on arrival on the other station, the deceleration to velocity zero is likewise instantaneous.

From [1], Eqt. (31), follows

$$N_{cz} = Q_x^c \ddot{Q}_y^c M^c$$

Thus

$$(15) \qquad n_{cz}(s) = M^c Q_x^c \ddot{q}_y^c(s)$$

See Figure 3 now. With $\dot{Q}_y^c$ = constant $\equiv$ K, as assumed previously, it is

$$(16) \qquad \dot{q}_y^c(s) = \frac{K}{s}$$

Because of $s\dot{q}(s) = \ddot{q}(s)$, $\ddot{q}^c_y(s)$ now becomes

$$(17) \qquad \ddot{q}^c_y(s) = K$$

which is in the time domain a δ - function at time zero. Therefore, (15) yields+)

$$(18) \qquad n_{cz}(s) = M^c Q^c_x K \equiv N_{cz}$$

which is also a δ - function. Now, $n_{cz}(s)$ has a positive sign if M^c moves from station 2 to station 1. For a positive $n_{cz}(s)$, the input $n_{cz}(s)$ becomes negative and vice versa. See (3) and [1], Eqts. (21) and (22).

The response R^m_{yc} for crew motion due to a δ - function input according to (18) becomes in the time domain [3]

$$(19) \qquad R^m_{yc} = - \frac{M^T E_x}{A_2 \omega} N_{cz}\, e^{-\frac{A_1}{2A_2} t} \sin \omega t$$

where

$$(20) \qquad \omega = \left| \left[\left(\frac{A_1}{2A_2}\right)^2 - \frac{A_0}{A_2} \right] \right|^{1/2}$$

and, especially, for $A_2 \simeq M^T I^b_{zz}$

$$(21) \qquad R^m_{yc} = - \frac{E_x}{I^b_{zz} \omega} N_{cz}\, e^{-\frac{C_1}{2M^T} t} \sin \omega t$$

+) Units of N_{cz} in (18) are ft.lb.sec.

where

$$(22) \qquad \omega = \left| \left[\left(\frac{C_1}{2M^T} \right)^2 - \frac{C_0}{M^T} \right] \right|^{1/2}$$

For a double δ - function, namely, acceleration at time $t = 0$, and deceleration at time $t = \tau$ to velocity zero, (19) becomes

$$(23) \qquad R^m_{yc} = - \frac{M^T E_x}{A_2 \omega} N_{cz} \left\{ e^{-\frac{A_1}{2A_2} t} \sin \omega t - e^{-\frac{A_1}{2A_2}(t-\tau)} \sin \omega (t-\tau) \right\}$$

$$\text{where } f(t-\tau) = \begin{cases} 0 \; ; \; t < \tau \\ f(t - \tau) \; ; \; t \geq \tau \end{cases}$$

and τ is the time it takes mass M^C to get from station to station.

For the case of having no damping, the amplitude of R^m_{yc} , $\hat{R}^m_{yc}$, is given as function of $\omega\tau$ for $t \geq \tau$

$$(24) \qquad |\hat{R}^m_{yc}| = \left| \frac{M^T E_x N_{cz}}{A_2 \omega} \left[2(1 - \cos \omega\tau) \right]^{1/2} \right|$$

which yields the maximum value

$$|\hat{R}^m_{yc}|_{max} = \left| 2 \frac{M^T E_x N_{cz}}{A_2 \omega} \right|$$

Eqt. (24) is plotted in Figure 4.

It can be shown that for the damped case, the maximum values of $\hat{R}^m_{yc}$ do not exceed those which are given in (24).

5. RESPONSE DURING ORBITAL MOTION

5.1 Evaluation of First Term in Eqt. (13)

First the functional relations of the external forces F^T_{ey}, F^b_{ey}, F^T_{ex}, and F^b_{ex} with respect to the orbital position of the cluster have to be established. See Figure 5. It describes the orientation of the carrier and the telescope during the orbital motion. The X^bY^b and X^TY^T coordinate axes lie in the orbital plane with the respective Z axes pointing upward. As the carrier vehicle is stablized in attitude with respect to external torques, it is assumed here for the following derivations that the coordinate axes have always the same orientation with respect to inertial space.

Four positions of the cluster are specially marked in Figure 5, namely, position I, II, III, and IV. They are spaced apart by the time T/4, where T is the time it takes to complete one orbit. The orbit is assumed to be circular, i.e., the c.m.+) performs a circular orbit about the earth.

The aforementioned forces are given in the following table:

Position	F^T_{ey}	F^b_{ey}	F^T_{ex}	F^b_{ex}
I	0	0	g^TM^T	g^bM^b
II	g^TM^T	g^bM^b	0	0
III	0	0	$-g^TM^T$	$-g^bM^b$
IV	$-g^TM^T$	$-g^bM^b$	0	0

where g^T and g^b denote the gravitational accelerations at (c.m.) and $(c.m.)_b$ which act in the directions of $\vec{R}^T$ and $\vec{R}^b$ respectively. The gravitational forces are the dominating external forces,

---

+) See [1], c.m. is center of mass of cluster.

and other external forces, e.g., aerodynamical forces, are neglected, as the main purpose of this memorandum is to compare the influence of the dominant forces with crew motion. For the following derivations c.m. and $(c.m.)_b$ shall be identical, furthermore, the two vectors $\vec{R}^T$ and $\vec{R}^b$ are, in general, not coinciding.

But the angle

$$\cos^{-1} \frac{\vec{R}^b \cdot \vec{R}^T}{|\vec{R}^b| \; |\vec{R}^T|}$$

between these vectors is so small - about some seconds of arc - that one can write, e.g., at positions II and IV

$$|F^b_{ey}| = g^b M^b \tag{25}$$

etc., as it was done in the above table.

Finally, as function of time, starting at position I, those forces are

$$\begin{aligned} F^T_{ey} &= g^T M^T \sin \Omega t \\ F^b_{ey} &= g^b M^b \sin \Omega t \\ F^T_{ex} &= g^T M^T \cos \Omega t \\ F^b_{ex} &= g^b M^b \cos \Omega t \end{aligned} \tag{26}$$

where Ω is the orbital period.

For the evaluation of (26), the values of g^T and g^b as functions of time, i.e., as functions of the orbital positions, must be established.

Because of the circular orbit, g^b is a constant, and

$$g^b = g_o \left(\frac{R_o}{R^b} \right)^2 \tag{27}$$

where g_o is the gravitational acceleration at the surface of the earth and R_o is the earth's radius.

Furthermore, it is also true that

$$g^T = g_o \left(\frac{R_o}{R^T} \right)^2 \tag{28}$$

The computation of R^T (orbital position) yields, if position I is the reference position,

$$R^T = R^b - \Delta R^T \sin(\Omega t + \Gamma) \tag{29}$$

where

$$\Delta R^T = [a_x{}^2 + a_y{}^2]^{1/2} \tag{30}$$

and

$$\Gamma = \frac{\pi}{2} - \sin^{-1} \left(\frac{a_y}{\Delta R^T} \right) \tag{31}$$

The terms a_x and a_y are represented through +)

(32)
$$a_x = Q_x + R^m_{x0}$$
$$a_y = Q_y + R^m_{y0} + O^T_y$$

Figure 6 shows R^T as a function of the orbital position. Using (29), the expression for g^T, Eqt. (28), becomes

(33)
$$g^T = g_o \left(\frac{R_o}{R^b}\right)^2 \left[1 + \frac{2\Delta R^T}{R^b} \sin\,(\Omega t + \Gamma)\right]$$

or

(34)
$$g^T = g_o \left(\frac{R_o}{R^b}\right)^2 \left\{1 + \frac{2\Delta R^T}{R^b} [\sin\Omega t \sin\alpha + \cos\Omega t \cos\alpha]\right\}$$

where

$$\alpha = \pi/2 - \Gamma$$

It follows from (26), (27), and (34)

(35)
$$F^T_{ey} = M^T g_o \left(\frac{R_o}{R^b}\right)^2 \left\{\sin\,\Omega t + \frac{\Delta R^T}{R^b} [(1-\cos 2\Omega t)\sin\alpha + \cos\alpha \sin 2\Omega t]\right\}$$

+) The components P_x and P_y are neglected in Figure 5, so is in (32), for the computation of a_y, the change in length $L^{-1}\{r^m_y(s)\}$ from (13). L^{-1} indicates the inverse Laplace-Transform.

$$F_{ey}^{b} = M^{b} g_{o} \left(\frac{R_{o}}{R^{b}}\right)^{2} \sin \Omega t \tag{36}$$

Therefore, [3]

$$\begin{aligned} & M^{b} f_{ey}^{T}(s) - M^{T} f_{ey}^{b}(s) \\ & = M^{b} M^{T} g_{o} \left(\frac{R_{o}}{R^{b}}\right)^{2} \left\{ \frac{\Delta R^{T}}{R^{b}} \left[\frac{(2\Omega)^{2} \sin\alpha}{s(s^{2}+(2\Omega)^{2})} + \frac{2\Omega\cos\alpha}{s^{2}+(2\Omega)^{2}} \right] \right\} \end{aligned} \tag{37}$$

Thus, for the first term in (13), the expression is obtained

$$\frac{I_{zz}^{b} M^{b} M^{T} g_{o} (R_{o})^{2} \Delta R^{T}}{A_{2}(M^{b}+M^{T})(R^{b})^{3}[s^{2} + \frac{A_{1}}{A_{2}}s + \frac{A_{o}}{A_{2}}]} \left\{ \frac{(2\Omega)^{2}\sin\alpha}{s(s^{2}+(2\Omega)^{2})} + \frac{2\Omega\cos\alpha}{s^{2}+(2\Omega)^{2}} \right\} \tag{38}$$

The result of the inverse transformation of (38) will be stated here in (39) for the case of having no damping, i.e., $A_1 = 0$, as this yields the term's largest value. Furthermore, it is assumed that $\Omega << \omega$ +)

$$\frac{I_{zz}^{b} M^{b} M^{T} g_{o} (R_{o})^{2} \Delta R^{T}}{A_{2}(M^{T}+M^{b})(R^{b})^{3}\omega^{2}} \left\{ \cos\alpha\sin 2\Omega t + \sin\alpha(1-\cos 2\Omega t) \right\} \tag{39}$$

where $\omega^2 = A_0/A_2$.

+) $\Omega = \omega$ should be prevented and $\Omega >> \omega$ can be omitted from the investigations here, because of the very low values of Ω, i.e., $\Omega \gtrsim 10^{-3}$ 1/sec. See Section 3.

The maximum value of the bracket term in (39) is 2. Thus, the maximum value of the first term in (13) would be

$$(40) \qquad \frac{I_{zz}^{b} M^{b} M^{T} g_{o} (R_{o})^{2} \Delta R^{T} 2}{A_{2} (M^{T} + M^{b}) (R^{b})^{3} \omega^{2}}$$

5.2 EVALUATION OF THIRD TERM IN EQT. (13)

It is

$$(41) \qquad F_{ex} = F_{ex}^{T} + F_{ex}^{b}$$

and with respect to the linearized expressions [1]+)

$$(42) \qquad f_{ex}(s) = L\left\{dF_{ex}\right\} = L\left\{F_{ex}(t) - F_{ex0}\right\}$$

where F_{ex0} is the value of this force at position I in orbit. Thus

$$(43) \qquad dF_{ex} = (M^{T} g^{T} + M^{b} g^{b})(\cos \Omega t - 1)$$

From Figure 7 follows

$$(44) \qquad N_{ez}^{T} = - O_{y}^{T} F_{ex}^{T}$$

+) L - symbol for Laplace transform.

Thus

$$(45) \qquad dN_{ez}^{T} = O_y^T M^T g^T [1 - \cos \Omega t]$$

One can show that

$$(46) \qquad n_z^T(s) + \frac{M^T O_y^T}{M^b + M^T} f_{ex}(s) \simeq L \left\{ O_y^T M^T [g^T - g^b][1 - \cos \Omega t] \right\}$$

The evaluation of (46) yields

$$(47) \qquad M^T O_y^T g_o \left(\frac{R_o}{R^b} \right)^2 \frac{\Delta R^T}{R^b} \left\{ - \frac{2\Omega \sin\alpha}{s^2 + (2\Omega)^2} - \frac{\cos\alpha}{s} - \frac{s\cos\alpha}{s^2 + (2\Omega)^2} + \frac{2\Omega \sin\alpha}{s^2 + \Omega^2} + \frac{2s\cos\alpha}{s^2 + \Omega^2} \right\}$$

In the time domain, the following expression is obtained for the third term in (13), assuming again $A_1 = 0$ and $\omega >> \Omega$

$$(48) \qquad \frac{(I_{zz}^T + M^T O_y^T Q_y^*)}{I_{zz}^T A_2 \omega^2} \frac{g_o (R_o)^2 \Delta R^T}{(R^b)^3} \left\{ \sin\alpha [2\sin\Omega t - \sin 2\Omega t] \; 2\cos\alpha [\cos\Omega t - \cos^2 \Omega t] \right\}$$

The sum of the terms in the bracket is always < 5. Therefore, the upper limit value of the third term in (13) becomes

$$(49) \qquad < \frac{(I^T_{zz} + M^T O^T_y Q^*_y) g_o (R_o)^2 \Delta R^T}{I^T_{zz} A_2 \omega^2 (R^b)^3} \, 5$$

A subsequent comparison of the maximum values of this term and the first term, Eqt. (40), yields

$$(50) \qquad \frac{\text{first term (13)}}{\text{third term (13)}} \approx \frac{I^b_{zz}}{M^T E_x O^T_y} \frac{2}{5}$$

The ratio (50) is usually much larger than 1. Thus, the first term exceeds by far the influence of the third. Therefore, for the response due to orbital motion, R^m_{yo} , only the first term in (13), Eqt. (29), must be considered.

6. ON DAMPING AND EIGENFREQUENCY

From (21), it follows for $A_1 = 0$ and $I^b_{zz} = 4 \times 10^6$ ft.lb.sec^2 that

$$(51) \qquad \left| \frac{\hat{R}^m_{yc}}{E_x N_{cz}} \right| = \frac{0.25 \times 10^{-6}}{\omega}$$

where $\hat{R}^m_{yc}$ is the maximum amplitude of R^m_{yc} . Eqt. (51) is plotted in Figure 8.

From (40) the following expression is obtained for the amplitude $\hat{R}^m_{yo}$ of the deflection for the undamped case which is due to the orbital motion

(52) $$\left|\hat{R}^m_{yo}\right| = \left|2\,\frac{g_o \Delta R^T}{\omega^2 (R_o + 3OH)}\right|$$

where $R^b = R_o + OH$, OH being the orbital height ($OH << R_o$)
$A_2 \approx M^T I^b_{zz}$, $M^T + M^b \approx M^b$

The ratio

(53) $$\left|\frac{\hat{R}^m_{yo}(R_o + 3OH)}{\Delta R^T}\right| = \frac{2g_o}{\omega^2}$$

is also plotted in Figure 8.

Figure 9 shows the damping coefficient C_1 as function of the ratio r, where

(54) $$r = \frac{e^{-\frac{C_1}{2M^T}t}}{1}$$

for M^T = 167 slugs, t = 7 seconds - time to move from station to station if $K \equiv \dot{Q}^c_y$ = 3 ft/sec.

The ratio (54) is, of course, the ratio which the amplitudes $|\hat{R}^m_{yc}|$ can attain according to (21) at the respective stations.

The Eigenfrequency ω of the suspension system will be, in general, chosen within the range

$$(55) \qquad \Omega \ll \omega \ll \omega_{0T}$$

where ω_{0T} is the lowest Eigenfrequency of the telescope. Values of $\omega < \Omega$ yield large values for R^m_{yc} and R^m_{yo}, see Figure 8. Also, e.g., for $\omega \approx 10^{-4}$ 1/sec, one would obtain $T \approx 2\pi$ hours.

Furthermore, according to (20) or (22), it is probably very difficult to adjust C_0 and C_1 in such a way that values $\omega < \Omega$ can be obtained. After having determined desirable values for ω and C_1 from Figures 8 and 9, respectively, the spring constant C_0 is, see (22)

$$(56) \qquad C_0 = M^T \left[\left(\frac{C_1}{2M^T} \right)^2 - \omega^2 \right]$$

7. DISCUSSION OF RESULTS

Impulsive type crew motion is assumed, namely, a simple wall push-off of a crew member and, secondly, a wall push-off which is followed by an instantaneous slowdown to velocity zero after this crew member has traversed the workshop. For the orbital motion, the disturbance inputs to the system are the external forces which act on telescope and carrier and the external torque which acts about the telescope's gimbals.

It is shown that the maximum relative translational motion of the telescope is a direct function of the Eigenfrequency of the telescope's suspension system. The amplitude of the relative motion due to crew motion is inversely proportional to the Eigenfrequency while the relative motion due to orbital motion inversely proportional to the square of the Eigenfrequency. This is shown in Eqts. (19), (21), (23) and (40), (49), (50). Relative values for these two components of the telescope's relative translational motion are plotted in Figure 8 as functions of the Eigenfrequency. For the convenience of the reader, the definitions of the variables of this figure are restated here: $\hat{R}^m_{yc}$ is the maximum amplitude of the telescope's translational motion relative to the carrier due to crew motion, $\hat{R}^m_{yo}$ is the maximum amplitude of the telescope's translational motion relative to the carrier due to orbital motion, R_o is the radius of the earth, OH is the orbital height of the cluster (carrier and telescope), ΔR^T is the distance between center of mass of telescope and center of mass of carrier, N_{cz} is the applied impulse due to crew motion and E_x is the nominal X component of the distance between the center of mass of the cluster and the center of mass of the telescope. It is further shown that the Eigenfrequency should be greater than the frequency of the orbital motion and, if possible, below the lowest elastic Eigenfrequency of the telescope. In any case, however, the maximum allowable amplitude of the telescope's relative motion establishes an additional lower bound on the selection of the range of the Eigenfrequency of the telescope's suspension system.

As an example, assume for the Eigenfrequency the value $\omega = 10^{-1}$ 1/sec which is about 100 times the frequency of the orbital motion and certainly less than the lowest elastic Eigenfrequency which can be expected for the telescope. Then from Figure 8 follows

$$\left| \frac{\hat{R}^m_{yc}}{E_x N_{cz}} \right| = 2.5 \times 10^{-6} \text{ 1/ft.lb.sec.}$$

and

$$\left| \frac{\hat{R}_{yo}(R_o + 3\ \mathrm{OH})}{\Delta R^T} \right| = 6.4 \times 10^{3} \text{ ft.}$$

If now N_{cz} = 180 ft.lb.sec. (from Eqt. (18); for a crew member with the mass M^c = 6 slugs who is moving at a speed of K = 3 ft/sec at a distance Q_x^c = 10 ft. through the workshop), E_x = 40 ft., R_o = 6400 km, OH = 180 naut.m. and ΔR^T = 50 ft., one obtains

$$\left|\hat{R}^m_{yc}\right| = 1.8 \times 10^{-2} \text{ ft.}$$

and

$$\left|\hat{R}^m_{yo}\right| = 1.3 \times 10^{-2} \text{ ft.}$$

Finally, this memorandum was motivated by the work of G. M. Anderson [4].

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[2] J. W. Schindelin, "Representation of Equations of Motion of Space Telescope Carriers by System Matrices." Bellcomm, Inc., forthcoming memorandum.

[3] G. Doetsch, "Anleitung zum praktischen Gebrauch der Laplace Transformation." R. Oldenbourg, München, 1956.

[4] G. M. Anderson, "Dynamic Distortion of a Gimbaled Telescope." Bellcomm, Inc., Technical Memorandum, TM-69-1022-3, May 21, 1969.

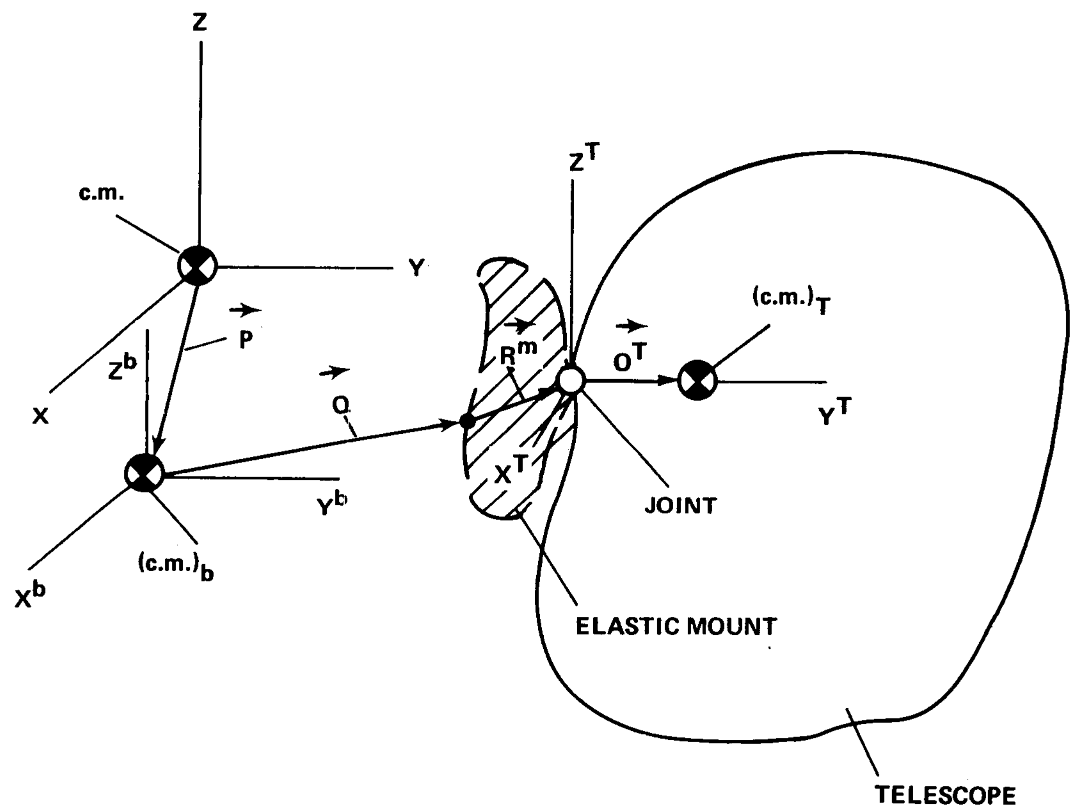

FIGURE 1 - REPRESENTATION OF THE CLUSTER, VECTOR AND COORDINATE SYSTEMS

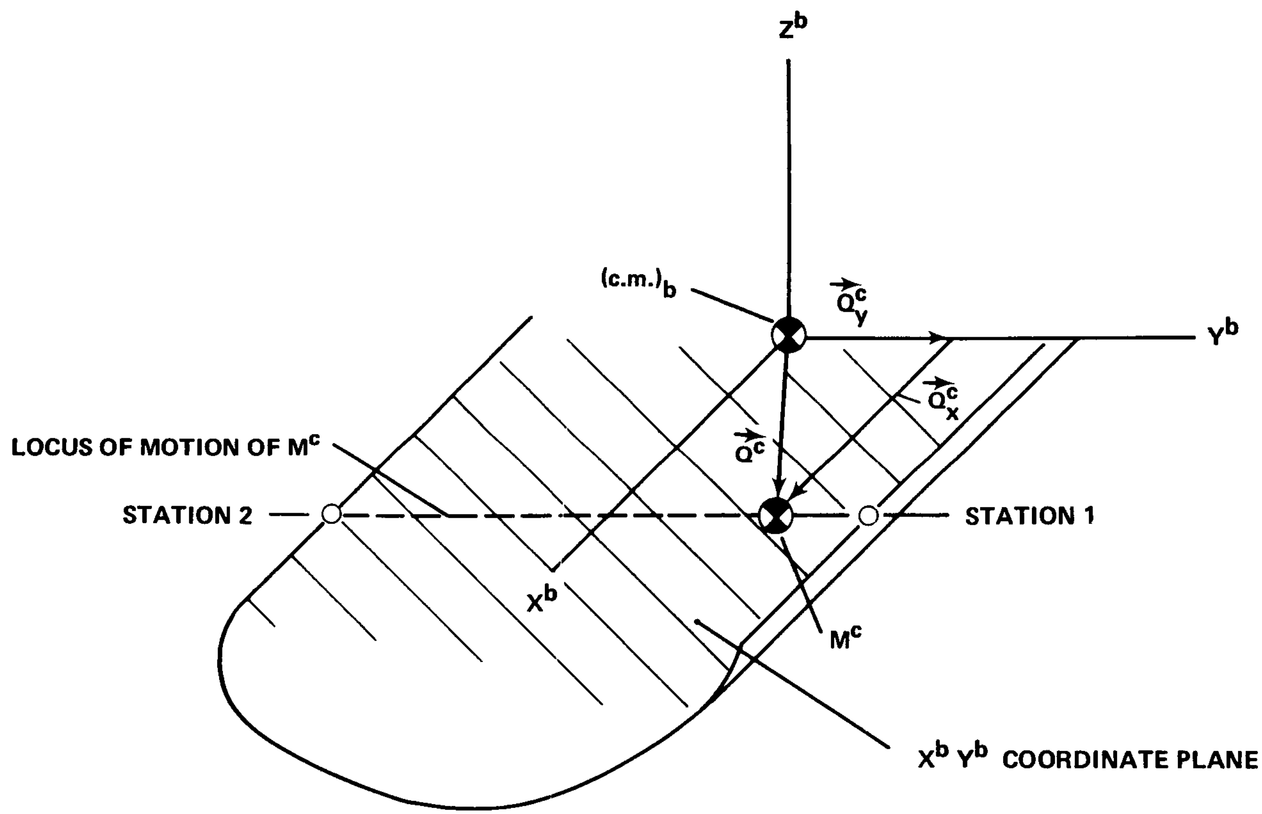

FIGURE 2 - WORKSHOP. REPRESENTATION OF LOCUS OF MOTION OF MASS M^c IN X^bY^b COORDINATE PLANE

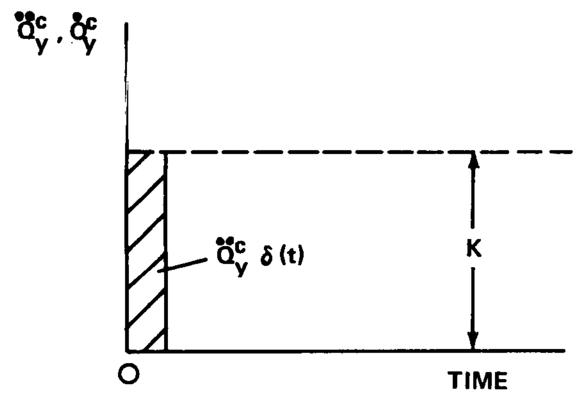

FIGURE 3 - ACCELERATION AND VELOCITY OF MASS M^c

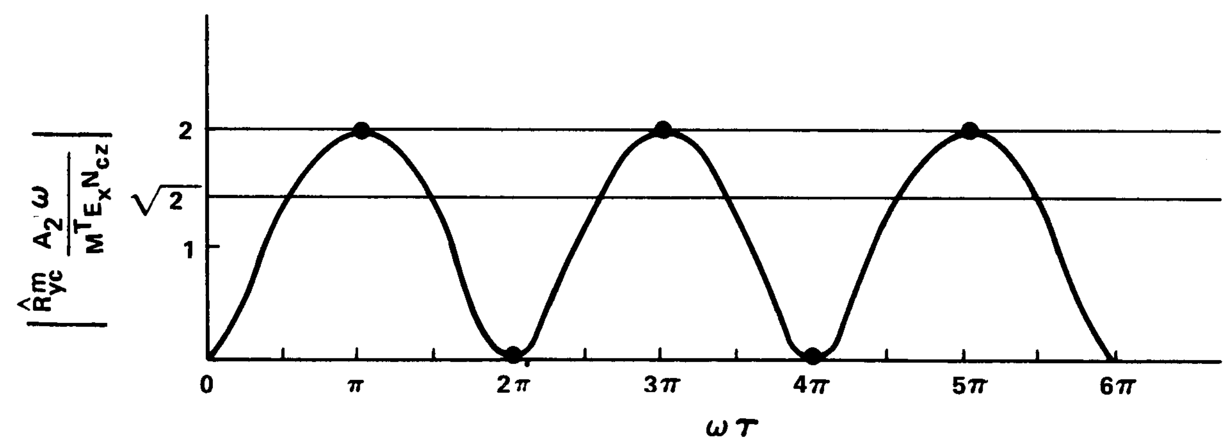

FIGURE 4 - MAXIMUM AMPLITUDE OF ELONGATION DUE TO CREW MOTION AS FUNCTION OF $\omega\tau$

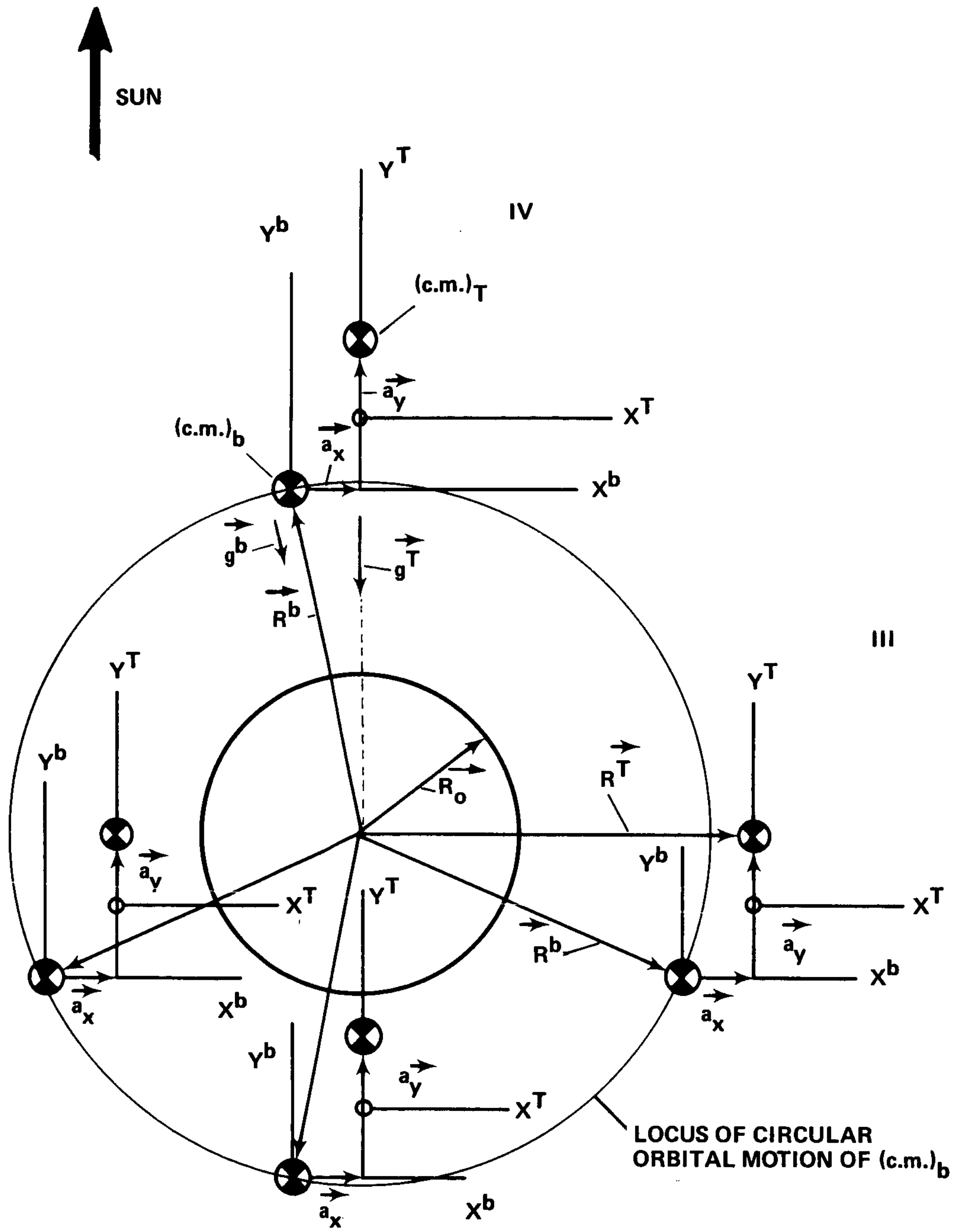

FIGURE 5 - ORIENTATION OF TELESCOPE CARRIER AND TELESCOPE DURING ORBITAL MOTION

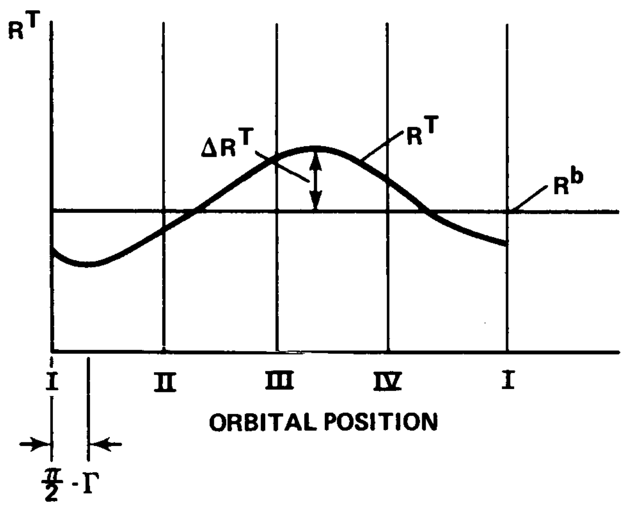

FIGURE 6 - VALUE OF R^T AS FUNCTION OF THE ORBITAL POSITION

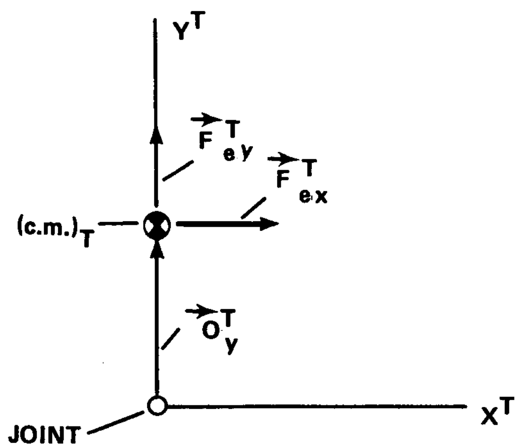

FIGURE 7 - CALCULATION OF TORQUE N_{ez}^{T} ABOUT THE JOINT

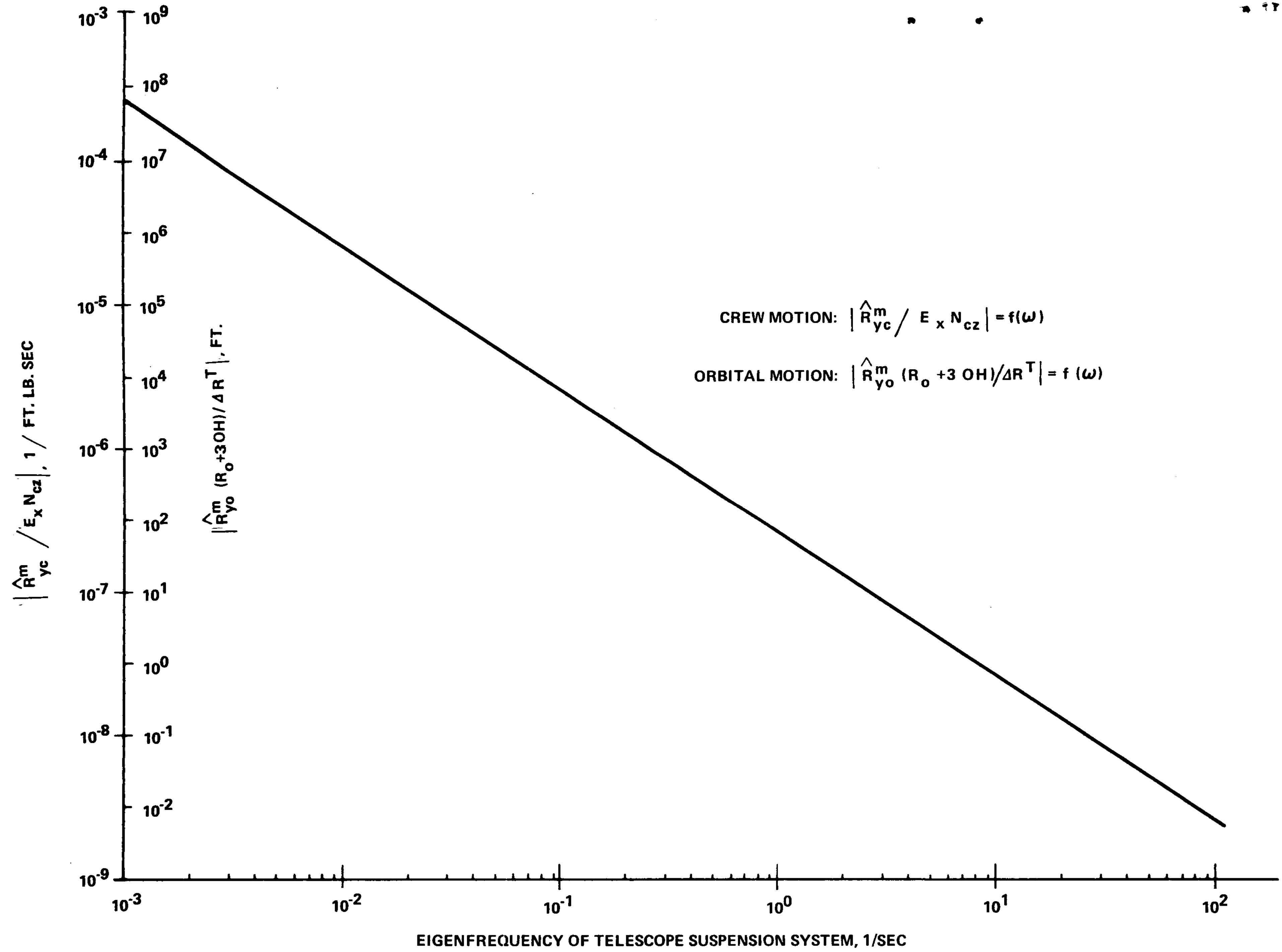

FIGURE 8 - MAXIMUM RELATIVE AMPLITUDES OF TELESCOPE TRANSLATIONAL MOTION VS. EIGENFREQUENCY

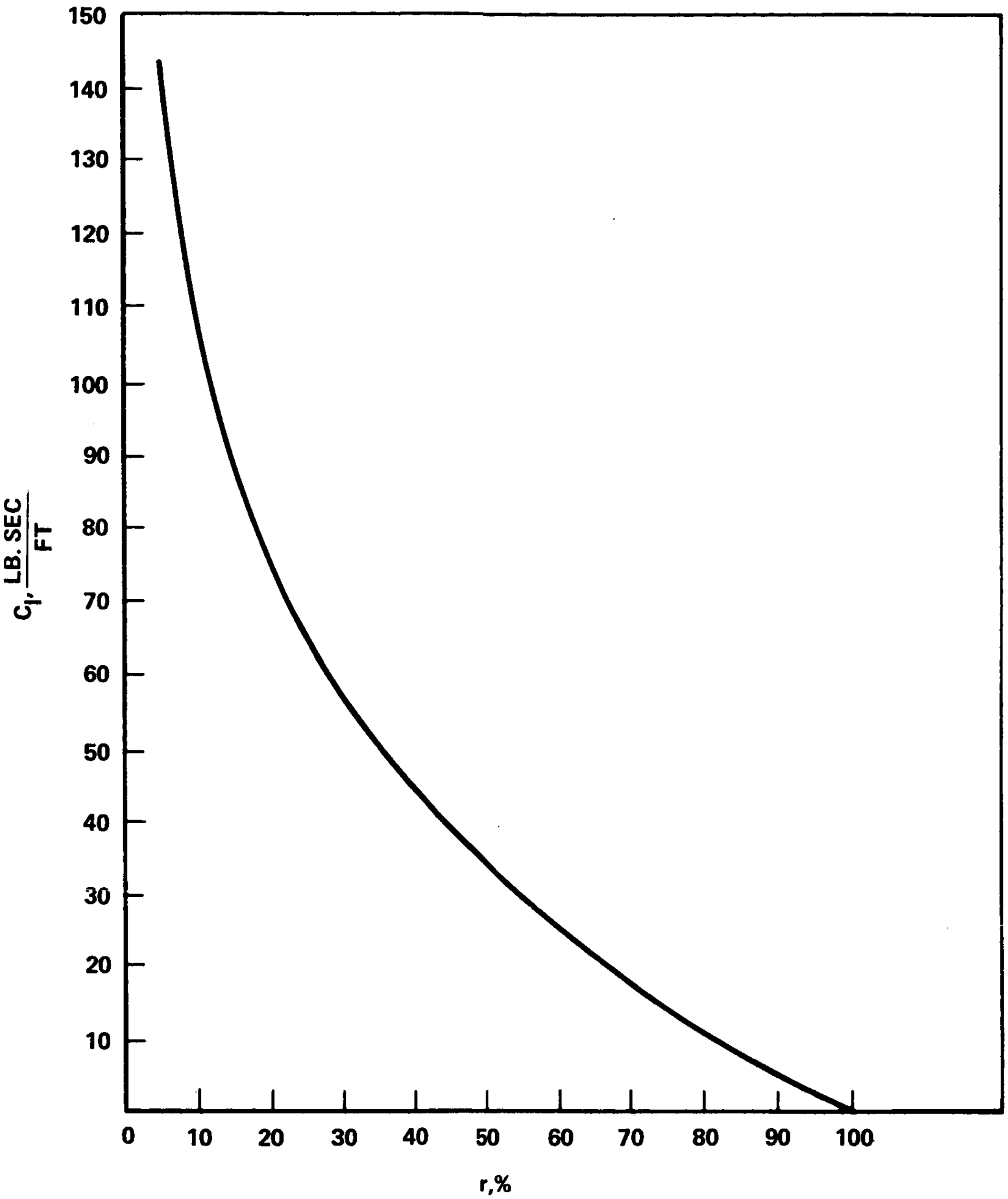

FIGURE 9 - DAMPING COEFFICIENT C_l VS. RATIO r